Massive Ghosts in Softly Broken SUSY Gauge Theories

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Abstract

It is shown that, due to soft supersymmetry breaking in gauge theories within the superfield formalism, there appears the mass for auxiliary gauge fields. It enters into the RG equations for soft masses of physical scalar particles and can be eliminated by solving its own RG equation. Explicit solutions up to the three-loop order in the general case and in the MSSM are given. The arbitrariness in choosing the initial condition is discussed.

1 Introduction

Recently, it has been realized [1, 2, 3] that renormalizations in a softly broken SUSY theory are not independent but follow from those of an unbroken SUSY theory. According to the approach advocated in Refs. [3, 4], one can perform the renormalization of a softly broken SUSY theory in the following straightforward way:

One takes renormalization constants of a rigid theory, calculated in some massless scheme, substitutes for the rigid couplings (gauge and Yukawa) their modified expressions, that depend on a Grassmannian variable, and expands over this variable.

This gives renormalization constants for the soft terms. Differentiating them with respect to a scale, one can find corresponding renormalization-group equations.

Thus, the soft-term renormalizations are not independent but can be calculated from the known renormalizations of a rigid theory with the help of the differential operators. Explicit form of these operators has been found in a general case and in some particular models like SUSY GUTs or the MSSM [3]. The same expressions have been obtained also in a somewhat different approach in Ref. [2, 5, 6].

There is, however, some minor difference. The authors of [2, 5] have used the component approach, while in [1, 3, 4], the superfield formalism is exploited. This creates the usual difference in gauge-fixing and ghost field terms and in the renormalization scheme. The latter is related to the choice of regularization. In [2, 5], the dimensional reduction (DRED) regularization is used. In this case, one is bounded to introduce the so-called ϵ-scalars to compensate the lack of bosonic degrees of freedom in 4-2ϵ

dimensions. These ϵ-scalars in due course of renormalization acquire a soft mass that enters into the RG equations for soft masses of physical scalar particles. This problem has been discussed in [7]. If one gets rid of the ϵ-scalar mass by changing the renormalization scheme, DRED $\to$ DRED$'$, there appears an additional term in RG equations for the soft scalar masses [8, 9] called X [5]. This term is absent in RG equations in Refs. [1, 3, 4].

We have to admit that, indeed in our approach, though the ϵ-scalars in the superfield formalism are absent, that term appears in higher orders and is related to the soft masses of other unphysical particles, the auxiliary gauge fields. We show below how it emerges in the superfield formalism and coincides with that of Ref. [5]. Thus, the two approaches finally merge.

2 Massive Auxiliary Fields

Consider an arbitrary $N=1$ SUSY gauge theory with unbroken SUSY within the superfield formalism. The Lagrangian of a rigid theory is given by

$$\begin{aligned}\mathcal{L}_{rigid} &= \int d^2\theta\, \frac{1}{4g^2}\mathrm{Tr}W^\alpha W_\alpha + \int d^2\bar\theta\, \frac{1}{4g^2}\mathrm{Tr}\bar W_{\dot\alpha}\bar W^{\dot\alpha} \\ &+ \int d^2\theta d^2\bar\theta\ \bar\Phi^i(e^V)^j_i\Phi_j + \int d^2\theta\ \mathcal{W} + \int d^2\bar\theta\ \bar{\mathcal{W}},\end{aligned} \tag{1}$$

where

$$W_\alpha = -\frac{1}{4}\bar D^2 e^{-V}D_\alpha e^V, \quad \bar W_{\dot\alpha} = -\frac{1}{4}D^2 e^{-V}\bar D_{\dot\alpha}e^V,$$

are the gauge field strength tensors and the superpotential $\mathcal{W}$ has the form

$$\mathcal{W} = \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}M^{ij}\Phi_i\Phi_j. \tag{2}$$

To fix the gauge, the usual gauge-fixing term can be introduced. It is useful to choose it in the form

$$\mathcal{L}_{gauge-fixing} = -\frac{1}{16}\int d^2\theta d^2\bar\theta \mathrm{Tr}\left(\bar f f + f\bar f\right) \tag{3}$$

where the gauge fixing condition is taken as

$$f = \bar D^2\frac{V}{\sqrt{\xi g^2}}\ , \quad \bar f = D^2\frac{V}{\sqrt{\xi g^2}}\ . \tag{4}$$

Then, the corresponding ghost term is [10]

$$\mathcal{L}_{ghost} = i\int d^2\theta\, \frac{1}{4}\mathrm{Tr}\ b\,\delta_c f - i\int d^2\bar\theta\, \frac{1}{4}\mathrm{Tr}\ \bar b\,\delta_{\bar c}\bar f, \tag{5}$$

where c and b are the Faddeev–Popov ghost and antighost chiral superfields, respectively, and δ_c is the gauge transformation with the replacement of gauge superfield parameters $\Lambda(\bar\Lambda)$ by chiral (antichiral) ghost fields $c(\bar c)$.

For our choice of the gauge-fixing condition, the gauge transformation of f looks like

$$\delta_\Lambda f = \bar D^2\delta_\Lambda\frac{V}{\sqrt{\xi g^2}} = i\bar D^2\frac{1}{\sqrt{\xi g^2}}\mathcal{L}_{V/2}[\Lambda+\bar\Lambda+\coth(\mathcal{L}_{V/2})(\Lambda-\bar\Lambda)], \tag{6}$$

where $\mathcal{L}_X Y \equiv [X,Y]$. Equation (5) then takes the form

$$\begin{aligned}
\mathcal{L}_{ghost} &= -\int d^2\theta \, \frac{1}{4}\mathrm{Tr}\; b\bar{D}^2 \frac{1}{\sqrt{\xi g^2}} \mathcal{L}_{V/2}[c+\bar{c}+\coth(\mathcal{L}_{V/2})(c-\bar{c})] + \quad h.c. \\
&= \int d^2\theta d^2\bar{\theta}\; \mathrm{Tr}\; \left(\frac{b+\bar{b}}{\sqrt{\xi g^2}}\right) \mathcal{L}_{V/2}[c+\bar{c}+\coth(\mathcal{L}_{V/2})(c-\bar{c})] \qquad (7) \\
&= \int d^2\theta d^2\bar{\theta}\; \mathrm{Tr}\; \left(\frac{b+\bar{b}}{\sqrt{\xi g^2}}\right) \left((c-\bar{c}) + \frac{1}{2}\Big[V,(c+\bar{c})\Big] + \frac{1}{12}\Big[V,\Big[V,(c-\bar{c})\Big]\Big] + \ldots\right).
\end{aligned}$$

The resulting Lagrangian together with the gauge-fixing and the ghost terms are invariant under the BRST transformations. For a rigid theory in our normalization of the fields, they have the form [10]

$$\begin{aligned}
&\delta V = \epsilon \mathcal{L}_{V/2}[c+\bar{c}+\coth(\mathcal{L}_{V/2})(c-\bar{c})], \\
&\delta\, c^a = -\frac{i}{2}\epsilon f^{abc} c^b c^c \;, \qquad \delta\, \bar{c}^a = -\frac{i}{2}\epsilon f^{abc} \bar{c}^b \bar{c}^c \;, \\
&\delta\, b^a = \frac{1}{8}\epsilon \bar{D}^2 \bar{f}^a \;, \quad \delta\, \bar{b}^a = \frac{1}{8}\epsilon D^2 f^a. \qquad (8)
\end{aligned}$$

To perform the SUSY breaking, that satisfies the requirement of "softness", one can introduce a gaugino mass term as well as cubic and quadratic interactions of scalar superpartners of the matter fields [11]

$$-\mathcal{L}_{soft-breaking} = \left[\frac{M}{2}\lambda\lambda + \frac{1}{6}A^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}B^{ij}\phi_i\phi_j + h.c.\right] + (m^2)^i_j \phi^*_i \phi^j, \quad (9)$$

where λ is the gaugino field, and ϕ_i is the lowest component of the chiral matter superfield.

One can rewrite the Lagrangian (9) in terms of N=1 superfields introducing the external spurion superfields [11] $\eta = \theta^2$ and $\bar{\eta} = \bar{\theta}^2$, where θ and $\bar{\theta}$ are Grassmannian parameters, as [1]

$$\begin{aligned}
\mathcal{L}_{soft} &= \int d^2\theta \, \frac{1}{4g^2}(1-2M\theta^2)\mathrm{Tr}W^\alpha W_\alpha + \int d^2\bar{\theta}\, \frac{1}{4g^2}(1-2\bar{M}\bar{\theta}^2)\mathrm{Tr}\bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}} \\
&+ \int d^2\theta d^2\bar{\theta}\;\; \bar{\Phi}^i(\delta^k_i - (m^2)^k_i \eta\bar{\eta})(e^V)^j_k \Phi_j \qquad (10) \\
&+ \int d^2\theta \left[\frac{1}{6}(y^{ijk} - A^{ijk}\eta)\Phi_i\Phi_j\Phi_k + \frac{1}{2}(M^{ij} - B^{ij}\eta)\Phi_i\Phi_j\right] + h.c.
\end{aligned}$$

Thus, one can interpret the soft terms as the modification of the couplings of a rigid theory. The couplings become external superfields depending on Grassmannian parameters θ and $\bar{\theta}$. To get the explicit expression for the modified couplings, consider eqs.(10). The first two terms give [3]

$$\frac{1}{g^2} \to \frac{1}{\tilde{g}^2} = \frac{1 - M\theta^2 - \bar{M}\bar{\theta}^2}{g^2}. \qquad (11)$$

Since the gauge field strength tensors W_α ($\bar{W}_\alpha$) are chiral (antichiral) superfields, they enter into the chiral (antichiral) integrands in eq.(10), respectively. Correspondingly, the

$M\theta^2$ term of eq.(11) contributes to the chiral integral, while the $\bar{M}\bar{\theta}^2$ term contributes to the antichiral one. There is no $\theta^2\bar{\theta}^2$ term in eq.(11), since it is neither chiral, no antichiral and gives no contribution to eq.(10).

However, modifying the gauge coupling in the gauge part of the Lagrangian, one has to do the same in the gauge-fixing (4) and ghost (7) parts in order to preserve the BRST invariance. Here one has the integral over the whole superspace rather than the chiral one. This means that if one adds to eq.(11) a term proportional to $\theta^2\bar{\theta}^2$, it gives a nonzero contribution. Moreover, even if this term is not added, it reappears as a result of renormalization.

We suggest the following modification of eq.(11)

$$\frac{1}{g^2} \to \frac{1}{\tilde{g}^2} = \frac{1 - M\theta^2 - \bar{M}\bar{\theta}^2 - \Delta\theta^2\bar{\theta}^2}{g^2}, \tag{12}$$

which gives the final expression for the soft gauge coupling

$$\tilde{g}^2 = g^2 \left(1 + M\theta^2 + \bar{M}\bar{\theta}^2 + 2M\bar{M}\theta^2\bar{\theta}^2 + \Delta\theta^2\bar{\theta}^2\right). \tag{13}$$

In our previous papers [3, 4], this Δ term was absent. It will be clear below that it is self-consistent to put $\Delta = 0$ in the lowest order of perturbation theory, but it appears in higher orders due to renormalizations.

One has to take into account, however, that, since the gauge-fixing parameter ξ may be considered as an additional coupling, it also becomes an external superfield and has to be modified. The soft expression can be written as

$$\tilde{\xi} = \xi \left(1 + x\theta^2 + \bar{x}\bar{\theta}^2 + (x\bar{x} + z)\theta^2\bar{\theta}^2\right), \tag{14}$$

where parameters x and z can be obtained by solving the corresponding RG equation (see Appendix A).

Having this in mind, we perform the following modification of the gauge fixing condition (4) first used in [12]

$$f \to \bar{D}^2 \frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}}, \quad \bar{f} \to D^2 \frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}}. \tag{15}$$

Then, the gauge-fixing term (3) becomes

$$\mathcal{L}_{gauge-fixing} = -\frac{1}{8}\int d^2\theta d^2\bar{\theta}\ \mathrm{Tr}\left(\bar{D}^2 \frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}} D^2 \frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}}\right), \tag{16}$$

This leads to the corresponding modification of the associated ghost term (5)

$$\mathcal{L}_{ghost} = \int d^2\theta d^2\bar{\theta}\ \mathrm{Tr}\ \frac{1}{\sqrt{\tilde{\xi}\tilde{g}^2}} \left(b + \bar{b}\right) \mathcal{L}_{V/2}[c + \bar{c} + \coth(\mathcal{L}_{V/2})(c - \bar{c})]. \tag{17}$$

To understand the meaning of the Δ term, consider the quadratic part of the ghost Lagrangian (17)

$$\mathcal{L}^{(2)}_{ghost} = \int d^2\theta d^2\bar{\theta}\ \mathrm{Tr}\ \frac{1}{\sqrt{\xi g^2}} \left(1 - \frac{1}{2}M\xi\theta^2 - \frac{1}{2}\bar{M}\xi\bar{\theta}^2 - \frac{1}{2}\Delta\xi\theta^2\bar{\theta}^2\right) \left(b + \bar{b}\right)(c - \bar{c})$$

$$
\begin{aligned}
&= \int d^2\theta d^2\bar\theta \ \mathrm{Tr}\ \frac{1}{\sqrt{\xi g^2}}\left(1-\frac{1}{2}\Delta\xi\theta^2\bar\theta^2\right)(b+\bar b)(c-\bar c) \\
&- \frac{1}{2}\int d^2\theta \ \mathrm{Tr}\ \frac{1}{\sqrt{\xi g^2}} M\xi bc + \frac{1}{2}\int d^2\bar\theta \ \mathrm{Tr}\ \frac{1}{\sqrt{\xi g^2}}\bar M\xi \bar b\bar c,
\end{aligned}
\tag{18}
$$

where we have used the explicit form of $\tilde\xi$ given in Appendix A.

If one compares this expression with the usual Lagrangian for the matter fields (10), one finds an obvious identification of the second line with the soft scalar mass term and the third line with the mass term in a superpotential. Thus, Δ plays the role of a soft mass providing the splitting in the ghost supermultiplet.

The other place where the Δ-term appears is the gauge-fixing term (16). Here it manifests itself as a soft mass of the auxiliary gauge field, one of the scalar components of the gauge superfield V.

To see this, consider the gauge-fixing term (16) in more detail. Expanding the vector superfield $V(x,\theta,\bar\theta)$ in components

$$
\begin{aligned}
V(x,\theta,\bar\theta) &= \mathbb{C}(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \frac{i}{2}\theta\theta N(x) - \frac{i}{2}\bar\theta\bar\theta\bar N(x) - \theta\sigma^\mu\bar\theta v_\mu(x) \\
&+ i\theta\theta\bar\theta[\bar\lambda(x) + \frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)] - i\bar\theta\bar\theta\theta[\lambda + \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)] + \frac{1}{2}\theta\theta\bar\theta\bar\theta[D(x) - \frac{1}{2}\Box\mathbb{C}(x)].
\end{aligned}
\tag{19}
$$

and substituting it into eq.(16) one finds

$$
\begin{aligned}
\mathcal{L}_{gauge-fixing} &= \frac{1}{2\xi g^2}\Big[-(D - \Box\mathbb{C} - \Delta\xi\mathbb{C} + \frac{i}{2}M\xi\bar N - \frac{i}{2}\bar M\xi N)^2 - (\partial^\mu v_\mu)^2 \\
&+ (\bar N - i\bar M\xi\mathbb{C})\Box(N + iM\xi\mathbb{C}) + 2i(\lambda + \frac{1}{2}\bar M\xi\chi)\sigma^\mu\partial_\mu(\bar\lambda + \frac{1}{2}M\xi\bar\chi) \\
&- 2(\lambda + \frac{1}{2}\bar M\xi\chi)\Box\chi - 2(\bar\lambda + \frac{1}{2}M\xi\bar\chi)\Box\bar\chi - 2i\Box\chi\sigma^\mu\partial_\mu\bar\chi\Big].
\end{aligned}
\tag{20}
$$

One can see from eq.(20) that the parameter M, besides being the gaugino soft mass, plays the role of a mass of the auxiliary field χ, while Δ is the soft mass of the auxiliary fields N and $\mathbb{C}$. All these fields are unphysical degrees of freedom of the gauge superfield. They are absent in the Wess-Zumino gauge, however, when the gauge-fixing condition is chosen in supersymmetric form (3), this gauge is no longer possible, and the auxiliary fields χ, N, and $\mathbb{C}$ survive. Thus, the extra Δ term is associated with unphysical, ghost, degrees of freedom, just like in the component approach, one has the mass of unphysical ϵ-scalars. When we go down with energy, all massive fields decouple, and we get the usual nonsupersymmetric Yang-Mills theory.

The Δ-term is renormalized and obeys its own RG equation which can be obtained from the corresponding expression for the gauge coupling via Grassmannian expansion. In due course of renormalization, this term is mixing with the soft masses of scalar superpartners and gives an additional term in RG equations for the latter (X term of Ref. [5] mentioned above).

At the end of this section, we would like to comment on the BRST invariance in a softly broken SUSY theory. The BRST transformations (8) due to our choice of normalization of the gauge and ghost fields do not depend on the gauge coupling. Hence, in a softly broken theory they remain unchanged. One can easily check that, despite the substitution $g^2 \to \tilde g^2$ and $\xi \to \tilde\xi$, the softly broken SUSY theory remains BRST invariant [12].

3 RG Equations for the Soft Parameters.

Thus, following the procedure described in Refs [3, 4], to get the RG equations for the soft terms, one has to modify the gauge (g_i^2) and Yukawa (y_{ijk}) couplings replacing them by external superfields :

$$\begin{aligned}
\tilde{g}_i^2 &= g_i^2(1 + M_i\eta + \bar{M}_i\bar{\eta} + (2M_i\bar{M}_i + \Delta_i)\eta\bar{\eta}), && (21)\\
\tilde{y}^{ijk} &= y^{ijk} - A^{ijk}\eta + \frac{1}{2}(y^{njk}(m^2)^i_n + y^{ink}(m^2)^j_n + y^{ijn}(m^2)^k_n)\eta\bar{\eta}, && (22)\\
\tilde{\bar{y}}_{ijk} &= \bar{y}_{ijk} - \bar{A}_{ijk}\bar{\eta} + \frac{1}{2}(y_{njk}(m^2)^n_i + y_{ink}(m^2)^n_j + y_{ijn}(m^2)^n_k)\eta\bar{\eta}. && (23)
\end{aligned}$$

Then, the β functions of RG equations for the soft masses of scalar superpartners of the matter fields and for the mass of the auxiliary gauge field are given by [3]

$$[\beta_{m^2}]^i_j = D_2\gamma^i_j, \qquad (24)$$

$$\beta_{\Sigma_{\alpha_i}} = D_2\gamma_{\alpha_i}, \qquad (25)$$

where γ^i_j and $\gamma_{\alpha_i} = \beta_{\alpha_i}/\alpha_i$ are the anomalous dimensions of the matter fields and of the gauge coupling, respectively, and we have introduced the notation

$$\Sigma_{\alpha_i} = M_i\bar{M}_i + \Delta_i.$$

The modified expression for the operator D_2 is

$$\begin{aligned}
D_2 &= \bar{D}_1 D_1 + \Sigma_{\alpha_i}\alpha_i\frac{\partial}{\partial\alpha_i} + \frac{1}{2}(m^2)^a_n\left(y^{nbc}\frac{\partial}{\partial y^{abc}} + y^{bnc}\frac{\partial}{\partial y^{bac}} + y^{bcn}\frac{\partial}{\partial y^{bca}}\right.\\
&\quad \left. + y_{abc}\frac{\partial}{\partial y_{nbc}} + y_{bac}\frac{\partial}{\partial y_{bnc}} + y_{bca}\frac{\partial}{\partial y_{bcn}}\right). \qquad (26)
\end{aligned}$$

It coincides now with that of Ref. [5] with $X_i = \Delta_i$.

To find Σ_{α_i}, one can use equation (25). In particular, using the expression for the anomalous dimension γ_α in case of a single non-abelian gauge group calculated up to three loops [13]

$$\begin{aligned}
\gamma_\alpha &= \alpha Q + 2\alpha^2 QC(G) - \frac{2}{r}\alpha\gamma^{i\,(1)}_j C(R)^j_i - \alpha^3Q^2C(G) + 4\alpha^3QC^2(G)\\
&\quad -\frac{6}{r}\alpha^3QC(R)^i_jC(R)^j_i - \frac{4}{r}\alpha^2C(G)\gamma^{i\,(1)}_j C(R)^j_i + \frac{3}{r}\alpha y^{ikm}y_{jkn}\gamma^{n\,(1)}_m C(R)^j_i\\
&\quad +\frac{1}{r}\alpha\gamma^{i\,(1)}_j\gamma^{j\,(1)}_p C(R)^p_i + \frac{6}{r}\alpha^2\gamma^{i\,(1)}_j C(R)^j_pC(R)^p_i, \qquad (27)
\end{aligned}$$

and the anomalous dimension of the matter field calculated up to two loops

$$\begin{aligned}
\gamma^i_j &= \frac{1}{2}y^{ikl}y_{jkl} - 2\alpha C(R)^i_j \qquad (28)\\
&\quad - (y^{imp}y_{jmn} + 2\alpha C(R)^p_j\delta^i_n)(\frac{1}{2}y^{nkl}y_{pkl} - 2\alpha C(R)^n_p) + 2\alpha^2QC(R)^i_j\ ,
\end{aligned}$$

one can get the solution

$$
\begin{aligned}
\Sigma_\alpha{}^{(1)} &= M^2, & (29)\\
\Sigma_\alpha{}^{(2)} &= \Delta_\alpha^{(2)} = -2\alpha[\frac{1}{r}(m^2)^i_j C(R)^j_i - M^2 C(G)], & (30)\\
\Sigma_\alpha{}^{(3)} &= \Delta_\alpha^{(3)} = \frac{\alpha}{2r}[\frac{1}{2}(m^2)^i_n y^{nkl} y_{jkl} + \frac{1}{2}(m^2)^n_j y^{ikl} y_{nkl} + 2(m^2)^m_n y^{ikn} y_{jkm} \\
&\quad + A^{ikl} A_{jkl} - 8\alpha M^2 C(R)^i_j] C(R)^j_i - 2\alpha^2 Q C(G) M^2 \\
&\quad - 4\alpha^2 C(G)[\frac{1}{r}(m^2)^i_j C(R)^j_i - M^2 C(G)]. & (31)
\end{aligned}
$$

These expressions for Δ_α (30,31) coincide with those obtained in Ref. [14] for the mass of the ϵ-scalars.

The nonzero Δ-term modifies the expression for the β function of the soft scalar mass starting from the second loop. Substituting eq.(30) into the expression for the differential operator D_2 gives in two loops

$$
\begin{aligned}
[\beta_{m^2}]^{i\,(2)}_j &= -(A^{ikp} A_{jkn} + \frac{1}{2}(m^2)^i_l y^{lkp} y_{jkn} + \frac{1}{2} y^{ikp} y_{lkn} (m^2)^l_j + \frac{2}{2} y^{ilp} (m^2)^s_l y_{jsn} \\
&+ \frac{1}{2} y^{iks} (m^2)^p_s y_{jkn} + \frac{1}{2} y^{ikp} (m^2)^s_n y_{jks} + 4\alpha m_A^2 C(R)^p_j \delta^i_n)(\frac{1}{2} y^{nst} y_{pst} - 2\alpha C(R)^n_p) \\
&- (y^{ikp} y_{jkn} + 2\alpha C(R)^p_j \delta^i_n)(\frac{1}{2} A^{nst} A_{pst} + \frac{1}{4}(m^2)^n_l y^{lst} y_{pst} + \frac{1}{4} y^{nst} y_{lst} (m^2)^l_p \\
&+ \frac{4}{4} y^{nlt} (m^2)^s_l y_{pst} - 4\alpha m_A^2 C(R)^n_p) + 12\alpha^2 m_A^2 Q C(R)^i_j \\
&- (A^{ikp} y_{jkn} - 2\alpha m_A C(R)^p_j \delta^i_n)(\frac{1}{2} y^{nst} A_{pst} + 2\alpha m_A C(R)^n_p) \\
&- (y^{ikp} A_{jkn} - 2\alpha m_A C(R)^p_j \delta^i_n)(\frac{1}{2} A^{nst} y_{pst} + 2\alpha m_A C(R)^n_p) \\
&+ 4\alpha^2 C(R)^i_j [\frac{1}{r}(m^2)^k_l C(R)^l_k - M^2 C(G)], \qquad (32)
\end{aligned}
$$

where the last term is an extra contribution due to nonzero Δ_α.

To argue that a solution for Δ_α exists in all orders of PT, one can consider the so-called NSVZ-scheme [15] where the anomalous dimension γ_α is equal to

$$
\gamma_\alpha^{NSVZ} = \alpha \frac{Q - 2r^{-1}\mathrm{Tr}[\gamma C(R)]}{1 - 2C(G)\alpha}. \qquad (33)
$$

Then the solution for Δ_α to all orders is

$$
\Delta_\alpha^{NSVZ} = -2\alpha \frac{r^{-1}\mathrm{Tr}[m^2 C(R)] - M^2 C(G)}{1 - 2C(G)\alpha}. \qquad (34)
$$

It coincides with X of Ref. [14].

This problem has been also addressed in Ref. [1], where originally the additional contribution to the soft term β function was absent. In a comment to paper [1] it is suggested that the discrepancy can be eliminated by introducing the term proportional to the mass of the ϵ-scalar in the superfield formalism

$$
\frac{\tilde{m}_A^2}{2} V_\mu^A V_\nu^A \hat{g}^{\mu\nu} = \frac{\tilde{m}_A^2}{2} \int d^4\theta \bar{\eta}\eta \frac{1}{16g^2} \bar{\sigma}_\mu^{\dot{\alpha}\alpha} \bar{D}_{\dot{\alpha}} (e^{-2gV} D_\alpha e^{2gV}) \bar{\sigma}_\nu^{\dot{\beta}\beta} \bar{D}_{\dot{\beta}} (e^{-2gV} D_\beta e^{2gV}) \hat{g}^{\mu\nu}, \quad (35)
$$

where $\hat{g}^{\mu\nu}$ is a 2ϵ-dimensional metric tensor.

Similar things were done in Ref. [16], where the appearance an extra term in RGE for the soft scalar masses is due to additional "evanescent" operator [17] in DRED scheme as in eq.(35). It leads to additional contribution in higher loops.

However, whenever it is true, technically, it is complicated. We propose here the other solution of this problem.

4 Illustration

As an illustration of the described procedure, we consider the case of the MSSM. Here instead of one there are three gauge couplings, and though the recipe is still the same, one faces some problem of the general nature. We obtain below the explicit solutions for the Σ_{α_i} terms that can be of interest for the applications in higher loops.

In the MSSM we have three gauge and three Yukawa couplings and, to simplify the formulas, we use the following notation

$$\alpha_i \equiv \frac{g_i^2}{16\pi^2}, \quad i=1,2,3; \quad Y_k \equiv \frac{y_k^2}{16\pi^2}, \quad k=t,b,\tau.$$

Then, the modified couplings (21-23) take the form

$$\begin{aligned} \tilde{\alpha}_i &= \alpha_i\left(1+M_i\eta+\bar{M}_i\bar{\eta}+(M_i\bar{M}_i+\Sigma_{\alpha_i})\eta\bar{\eta}\right), \qquad (36)\\ \tilde{Y}_k &= Y_k\left(1-A_k\eta-\bar{A}_k\bar{\eta}+(A_k\bar{A}_k+\Sigma_k)\eta\bar{\eta}\right), \qquad (37)\end{aligned}$$

where Σ_k is the sum of the soft masses squared corresponding to a given Yukawa vertex

$$\Sigma_t=\tilde{m}_Q^2+\tilde{m}_U^2+m_{H_2}^2, \quad \Sigma_b=\tilde{m}_Q^2+\tilde{m}_D^2+m_{H_1}^2, \quad \Sigma_\tau=\tilde{m}_L^2+\tilde{m}_E^2+m_{H_1}^2.$$

Now the RG equations for a rigid theory can be written in a universal form

$$\dot{a}_i=a_i\gamma_i(a), \qquad a_i=\{\alpha_i, Y_k\}, \qquad (38)$$

where $\gamma_i(a)$ stands for a sum of corresponding anomalous dimensions. In the same notation, the soft terms (36,37) take the form

$$\tilde{a}_i=a_i(1+m_i\eta+\bar{m}_i\bar{\eta}+S_i\eta\bar{\eta}), \qquad (39)$$

where $m_i=\{M_i,-A_k\}$ and $S_i=\{M_i\bar{M}_i+\Sigma_{\alpha_i}, A_k\bar{A}_k+\Sigma_k\}$.

Substituting eq.(39) into eq.(38) and expanding over η and $\bar{\eta}$, one can get the RG equations for the soft terms

$$\dot{\tilde{a}}_i=\tilde{a}_i\gamma_i(\tilde{a}). \qquad (40)$$

Consider first the F-terms. Expanding over η, one has

$$\dot{m}_i=\gamma_i(\tilde{a})|_F=\sum_j a_j\frac{\partial\gamma_i}{\partial a_j}m_j. \qquad (41)$$

This is just the RG equation for the soft terms M_i and A_k [2, 3]. Proceeding in the same way for the D-terms, one gets after some algebra

$$\dot{S}_i=2m_i\sum_j a_j\frac{\partial\gamma_i}{\partial a_j}m_j+\sum_j a_j\frac{\partial\gamma_i}{\partial a_j}S_j+\sum_{j,k}a_ja_k\frac{\partial^2\gamma_i}{\partial a_j\partial a_k}m_jm_k. \qquad (42)$$

Substituting $S_i = m_i\bar{m}_i + \Sigma_i$, one has the following RG equation for the mass terms

$$\dot{\Sigma}_i = \gamma_i(\tilde{a})|_D = \sum_j a_j \frac{\partial\gamma_i}{\partial a_j}(m_j m_j + \Sigma_j) + \sum_{j,k} a_j a_k \frac{\partial^2\gamma_i}{\partial a_j \partial a_k} m_j m_k. \qquad (43)$$

Using the explicit form of anomalous dimensions calculated up to some order, one can reproduce the desired RG equations for the soft terms. In case of squark and slepton masses, they contain the contributions from unphysical masses Σ_{α_i}. To eliminate them, one has to solve the equation for Σ_{α_i}. In the case of the MSSM up to three loops, the solutions are

$$\begin{aligned} \Sigma_{\alpha_1} &= M_1^2 - \alpha_1\sigma_1 - \frac{199}{25}\alpha_1^2 M_1^2 - \frac{27}{5}\alpha_1\alpha_2 M_2^2 - \frac{88}{5}\alpha_1\alpha_3 M_3^2 \\ &+ \frac{13}{5}\alpha_1 Y_t(\Sigma_t + A_t^2) + \frac{7}{5}\alpha_1 Y_b(\Sigma_b + A_b^2) + \frac{9}{5}\alpha_1 Y_\tau(\Sigma_\tau + A_\tau^2), \qquad (44) \\ \Sigma_{\alpha_2} &= M_2^2 - \alpha_2(\sigma_2 - 4M_2^2) - \alpha_2^2(4\sigma_2 + 9M_2^2) - \frac{9}{5}\alpha_2\alpha_1 M_1^2 - 24\alpha_2\alpha_3 M_3^2 \\ &+ 3\alpha_2 Y_t(\Sigma_t + A_t^2) + 3\alpha_2 Y_b(\Sigma_b + A_b^2) + \alpha_2 Y_\tau(\Sigma_\tau + A_\tau^2), \qquad (45) \\ \Sigma_{\alpha_3} &= M_3^2 - \alpha_3(\sigma_3 - 6M_3^2) - \alpha_3^2(6\sigma_3 - 22M_3^2) - \frac{11}{5}\alpha_3\alpha_1 M_1^2 - 9\alpha_3\alpha_2 M_2^2 \\ &+ 2\alpha_3 Y_t(\Sigma_t + A_t^2) + 2\alpha_3 Y_b(\Sigma_b + A_b^2), \qquad (46) \end{aligned}$$

where we have used the combinations [8]

$$\begin{aligned} \sigma_1 &= \frac{1}{5}\left[3(m_{H_1}^2 + m_{H_2}^2) + 3(\tilde{m}_Q^2 + 3\tilde{m}_L^2 + 8\tilde{m}_U^2 + 2\tilde{m}_D^2 + 6\tilde{m}_E^2)\right], \qquad (47) \\ \sigma_2 &= m_{H_1}^2 + m_{H_2}^2 + 3(3\tilde{m}_Q^2 + \tilde{m}_L^2), \qquad (48) \\ \sigma_3 &= 3(2\tilde{m}_Q^2 + \tilde{m}_U^2 + \tilde{m}_D^2). \qquad (49) \end{aligned}$$

Notice, however, that the solutions (47-49) correspond to particular boundary conditions, while, in general, one can use arbitrary ones. Here we encounter the general problem that the solutions for physical masses depend on the unphysical parameter (ϵ-scalar mass in the component approach in the DRED scheme and an auxiliary field mass Δ in the superfield approach).

The solution to this paradox, mentioned also in Ref. [7], follows from the observation that the running soft masses that obey the RG equations are not, strictly speaking, the observables and are scheme-dependent. More appropriate are the pole masses, that are scheme-independent. The authors of Ref. [7] proposed the solution of the paradox by passing to the DRED′ scheme via the shift of the running soft mass, which allows one to get rid of the unwanted ϵ-scalar mass and does not influence the pole mass. In one-loop order, the shift is

$$(m^2)_i^j|_{\overline{DR}'} = (m^2)_i^j|_{\overline{DR}} - \frac{2g_A^2 C_A(i)}{(4\pi)^2}\delta_i^j \tilde{m}_\epsilon^2, \qquad (50)$$

where $\tilde{m}_\epsilon$ is the ϵ-scalar mass. The procedure can be continued in the same way in higher loops.

One can easily see how a similar trick works in our approach in case of one gauge coupling (and, consequently, one Δ term). Indeed, consider eq.(43). It is a linear inhomogeneous differential equation. Hence, to any given solution of this equation one

can add an arbitrary solution of a homogeneous equation. In our case, the solution of a homogeneous equation is

$$\Sigma_i = \mathcal{C}\gamma_i, \quad i = \alpha_1, \alpha_2, \alpha_3, t, b, \tau, \tag{51}$$

where $\mathcal{C}$ is an overall constant.

Hence, if one has the only gauge coupling one can choose the constant $\mathcal{C}$ so that one can get any desirable boundary condition for Σ_α. The price for this is extra terms in the other Σ's (and soft masses) proportional to the corresponding anomalous dimensions. However, the shift of the running mass by a term proportional to the anomalous dimension does not change the pole mass, since it can be absorbed into the scale redefinition. This is due to the fact that the coefficient of the $\log\mu^2$ term is just the anomalous dimension of the field.

Thus, the arbitrariness in the unphysical mass boundary condition does not influence the physical masses.

However, one has only one overall constant $\mathcal{C}$, and the above argument clearly works when one has only one gauge coupling. In case of many couplings, it is more tricky, and we have not found an obvious explanation.

5 Conclusion

Summarizing, we would like to stress once again that soft breaking of supersymmetry can be realized via interaction with an external superfield that develops nonzero v.e.v.'s for its F and D components. In the superfield notation, it can be reformulated as a modification of the rigid couplings that become external superfields. The same is true for the gauge-fixing parameter that can also be considered as a rigid coupling. The soft masses of scalar particles obtain their contribution from the D-components of external superfields. The latter also lead to nonzero masses for unphysical degrees of freedom, ghost and gauge auxiliary fields. These unphysical masses enter into the RG equations for the physical scalars and have to be eliminated. This creates an ambiguity in the running scalar masses; it can be resolved by passing to the pole masses.

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Appendix A

The RG equation for the parameter ξ in a rigid theory is

$$\dot{\xi} = -\gamma_V \xi, \tag{A.1}$$

where γ_V is the anomalous dimension of the gauge superfield. To find the soft terms $x, \bar{x}$ and z, one should solve the modified equation

$$\dot{\tilde{\xi}} = -\gamma_V(\tilde{\alpha}, \tilde{y}, \tilde{\xi})\tilde{\xi}. \tag{A.2}$$

In one-loop order $\gamma_V = (b_1 + b_2\xi)\alpha$, where $b_1 + b_2 = Q$, and the solutions are

$$x = -(M + x_0)\frac{b_1 + b_2\xi}{Q}, \quad \bar{x} = -(\bar{M} + \bar{x}_0)\frac{b_1 + b_2\xi}{Q}, \tag{A.3}$$

$$z = -(\Sigma_\alpha + z_0)\frac{b_1 + b_2\xi}{Q} + \frac{b_2\xi}{Q}(M + x_0)(\bar{M} + \bar{x}_0)\frac{b_1 + b_2\xi}{Q}, \tag{A.4}$$

where $x_0, \bar{x}_0$, and z_0 are arbitrary constants. In the Abelian case when $b_1 = Q,\ b_2 = 0$, the solutions are simplified and can be chosen as

$$x = -M(1-\xi), \quad \bar{x} = -\bar{M}(1-\xi), \quad z = -\Sigma_\alpha(1-\xi) - M\bar{M}\xi(1-\xi).$$

Together with the expression for $\tilde{\alpha}$ (13) it gives eq.(18) above.

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